PART 1: QUESTIONS

Name:______________________________ **Age:**_____ **Id:**_______________ **Course:**__________

Algebra I - Exam 2 **Lesson: 4-6**

Instructions:

- Please begin by printing your Name, your Age, your Student Id , and your Course Name in the box above and in the box on the solution sheet.
- You have 90 minutes (class period) for this exam.
- You can not use any calculator, computer, cellphone, or other assistance device on this exam. However, you can set our flag to ask permission to consult your own one two-sided-sheet notes at any point during the exam (You can write concepts, formulas, properties, and procedures, but questions and their solutions from books or previous exams are not allowed in your notes).
- Each multiple-choice question is worth 5 points and each extra essay-question is worth from 0 to 5 points. (Even a simple related formula can worth some points).
- Set up your flag if you have a question.
- Relax and use strategies to improve your performance.

Exam Strategies to get the best performance:

- Spend 5 minutes reading your exam. Use this time to classify each Question in (E) Easy, (M) Medium, and (D) Difficult.
- Be confident by solving the easy questions first then the medium questions.
- Be sure to check each solution. In average, you only need 30 seconds to test it. (Use good sense).
- Don't waste too much time on a question even if you know how to solve it. Instead, skip the question and put a circle around the problem number to work on it later. In average, the easy and medium questions take up half of the exam time.
- Solving the all of the easy and medium question will already guarantee a minimum grade. Now, you are much more confident and motivated to solve the difficult or skipped questions.
- Be patient and try not to leave the exam early. Use the remaining time to double check your solutions.

1. Solve:

$x - 2(5 - x) = -4$

a) 1
b) 2
c) 3
d) 4
e) There is no solution.

Solution: b

$x - 2(5 - x) = -4$
$x - 10 + 2x = -4$
$3x = -4 + 10$
$3x = 6$
$x = \dfrac{6}{3}$
$x = 2$

2. Solve:

$$\frac{5x - 4}{3} = \frac{10 - 3x}{2}$$

a) 1
b) 2
c) 3
d) 4
e) There is no solution.

Solution: b

$\dfrac{5x - 4}{3} = \dfrac{10 - 3x}{2}$
$2(5x - 4) = 3(10 - 3x)$
$10x - 8 = 30 - 9x$
$10x + 9x = 30 + 8$
$19x = 38$
$x = \dfrac{38}{19}$
$x = 2$

3. George wants to work 5 hours per day in a barber shop for $9 an hour. A total of 25% of his salary goes towards his dog, Philipe. How many days must he work to save $270 for a new pair of scissors?

a) 2 days
b) 4 days
c) 6 days
d) 8 days
e) He can't afford the new pair of scissors.

Solution: d

x = number of hours.
d = number of days.
If 25% of his salary is deducted then he makes only 75% or $\dfrac{3}{4}$ of the total amount paid.
Equation: $\dfrac{3}{4}$ * Total amount paid = 270
$9x * \dfrac{3}{4} = 270$
$9x = \dfrac{270}{\frac{3}{4}}$
$9x = \dfrac{270}{1} * \dfrac{4}{3}$
$9x = \dfrac{90}{1} * \dfrac{4}{1}$
$9x = 360$
$x = \dfrac{360}{9}$
$x = 40$ hours
$d = \dfrac{x}{5}$
$d = 8$ days

4. An exam of 60 questions has the following rules:

- 10 points for each correct answer.
- -3 points for each incorrect answer.

How many correct answers do you need to get 340 points on the exam?

a) 10 correct answers.
b) 20 correct answers.
c) 30 correct answers.
d) 40 correct answers.
e) None of the above.

Solution: d

x = number of correct answers.
$10x - 3(60 - x) = 340$
$10x - 180 + 3x = 340$
$13x = 520$
$x = 40$ correct answers

5. Solve:

$$\frac{x-2}{x+1} - \frac{x-3}{x-1} = \frac{2}{x^2-1} \; ; x \in \mathbb{R}$$

a) 0
b) 1
c) 2
d) 3
e) There is no solution.

Solution: d

Existence Condition: $x \neq \pm 1$.

$$\frac{(x-2)(x-1)-(x-3)(x+1)}{(x+1)(x-1)} - \frac{x-3}{x-1} = \frac{2}{(x+1)(x-1)}$$

$x^2 - x - 2x + 2 - x^2 - x + 3x + 3 = 2$
$-x + 5 = 2$
$-x = 2 - 5$
$-x = -3$
$x = \frac{-3}{-1}$
$x = 3$

6. Solve:

$$\frac{x}{x-2} + 2x = \frac{2x^2-6}{x-2} \; ; x \in \mathbb{R}$$

a) There are infinite solutions.
b) There are only one solution
c) There is no solution..
d) There are two solutions.
e) There are three solutions.

Solution: c

Existence Condition: $x \neq 2$
$x + 2x(x-2) = 2x^2 - 6$
$x + 2x^2 - 4x = 2x^2 - 6$
$-3x = -6$
$x = \frac{-6}{-3}$
$x = 2$ (Discarded).

Thus, there is no solution.

7. Let x_1 and x_2 be the solutions of the equation, $ax^2 + bx + c = 0 \; ; a \neq 0$.

I. $x_1 . x_2 = \frac{b}{a}$

II. $x_1 . x_2 = \frac{c}{a}$

III. $ax^2 + bx + c = a(x + x_1)(x + x_2)$

Then:

a) Only I is true.
b) Only II is true.
c) Only III is true.
d) Only I and II are true.
e) I,II, and III are true.

Solution: b

Direct application of formulas.

I - False, $x_1 + x_2 = \frac{-b}{a}$
II - True.
III - False, $ax^2 + bx + c = a(x - x_1)(x - x_2)$

8. $ax^2 + bx + c = 0$ where a, b, and c are real constants with $a \neq 0$ and x is a variable such that $x \in \mathbb{R}$.

I. This equation could have no solution.
II. This equation could have three distinct solutions.
III. This equation could have two distinct solutions.
IV. This equation could have one distinct solution.

a) I is false.
b) II is false.
c) III is false.
d) IV is false.
e) None of the above.

Solution: b

The Quadratic formula is $x = \frac{-b \pm \sqrt{\Delta}}{2a}$ where $\Delta = b^2 - 4ac$ and $a \neq 0$.

If $\Delta < 0 \Rightarrow$ the equation has no solution.

If $\Delta = 0 \Rightarrow$ the equation has one distinct solution.

If $\Delta > 0 \Rightarrow$ the equation has two distinct solutions.

This makes II the only one that is incorrect.

9. The solutions of the equation, $x^2 - 3x + 2 = 0$ are:

a) $x_1 = 1$ and $x_2 = 2$
b) $x_1 = 2$ and $x_2 = 3$
c) $x_1 = 3$ and $x_2 = 4$
d) $x_1 = 4$ and $x_2 = 5$
e) None of the above.

Solution: a

$x^2 - 3x + 2 = 0$ ($a = 1,\ b = -3$, and $c = 2$)

$$S = \frac{-b}{a} \Rightarrow S = \frac{-(-3)}{1} \Rightarrow S = 3$$

$$P = \frac{c}{a} \Rightarrow P = \frac{2}{1} \Rightarrow P = 2$$

Quick test:

+1 +2 $\quad x_1 = 1$ or $x_2 = 2$

Using Quadratic Formula:

$\Delta = b^2 - 4ac$
$\Delta = (-3)^2 - 4(1)(2)$
$\Delta = 9 - 8$
$\Delta = 1$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}\ ; a \neq 0$$

$$x = \frac{-(-3) \pm \sqrt{1}}{2(1)}$$

$$x = \frac{3 \pm 1}{2} \Rightarrow \quad x_1 = 1 \text{ or } x_2 = 2$$

Note that $S = x_1 + x_2 = 3$ and $P = x_1 . x_2 = 2$.

10. Which equation below has solutions $x_1 = -6$ and $x_2 = 2$

a) $x^2 + 4x + 12 = 0$
b) $x^2 - 4x + 12 = 0$
c) $x^2 + 2x - 12 = 0$
d) $x^2 - 2x - 12 = 0$
e) None of the above.

Solution: e

$S = x_1 + x_2 \Rightarrow S = -6 + 2 \Rightarrow S = -4$

$P = x_1 . x_2 \Rightarrow P = (-6) * 2 \Rightarrow P = -12$

The Professor's formula is:

$x^2 - Sx + P = 0$
$x^2 - (-4)x + 12 = 0$
$x^2 + 4x + 12 = 0$

or $ax^2 + bx + c = a(x - x_1)(x - x_2)$ then for $a = 1$ we have:
$(x - (-6))(x - 2) = 0$
$(x + 6)(x - 2) = 0$
$x^2 - 2x + 6x - 12 = 0$
$x^2 + 4x - 12 = 0$

11. The solutions of $x^2 - 2x - 10 = 0$ are

a) $x_1 = 1 - \sqrt{2}$ and $x_2 = 1 + \sqrt{2}$
b) $x_1 = 1 - \sqrt{5}$ and $x_2 = 1 + \sqrt{5}$
c) $x_1 = 1 - \sqrt{7}$ and $x_2 = 1 + \sqrt{7}$
d) $x_1 = 1 - \sqrt{11}$ and $x_2 = 1 + \sqrt{11}$
e) None of the above.

Solution: d

$x^2 - 2x - 10 = 0$

$$S = \frac{-b}{a} \Rightarrow S = \frac{-(-2)}{1} \Rightarrow S = 2$$

$$P = \frac{c}{a} \Rightarrow P = \frac{(-10)}{1} \Rightarrow P = -10$$

$\Delta = b^2 - 4ac$
$\Delta = (-2)^2 - 4(1)(-10)$
$\Delta = 4 + 40$
$\Delta = 44$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}\ ; a \neq 0$$

$$x = \frac{-(-2) \pm \sqrt{44}}{2(1)}$$

$$x = \frac{2 \pm 2\sqrt{11}}{2}$$

$x = 1 \pm \sqrt{11} \qquad x_1 = 1 - \sqrt{11}$ or $x_2 = 1 + \sqrt{11}$

Note that $S = x_1 + x_2 = 2$ and $P = x_1 . x_2 = -10$

12. Given $x^2 - 4x - 1 = 0$, the Sum (S) and the Product (P) of the solutions are

a) $S = 4$ and $P = 1$
b) $S = 1$ and $P = 4$
c) $S = 4$ and $P = -1$
d) $S = -4$ and $P = -1$
e) None of the above.

Solution: c

$x^2 - 4x - 1 = 0 \quad (a = 1, b = -4, \text{ and } c = -1)$

$S = \dfrac{-b}{a} \Rightarrow S = \dfrac{-(-4)}{1} \Rightarrow S = 4$

$P = \dfrac{c}{a} \Rightarrow P = \dfrac{-1}{1} \Rightarrow P = -1$

13. Let x_1,x_2, and x_3 be the solutions of the equation $x^3 - 3x^2 + 2x = 0$. Then:

a) $x_1 + x_2 + x_3 = -1$
b) $x_1 + x_2 + x_3 = 0$
c) $x_1 + x_2 + x_3 = 1$
d) $x_1 + x_2 + x_3 = 2$
e) $x_1 + x_2 + x_3 = 3$

Solution: e

$x^3 - 3x^2 + 2x = 0$
$x(x^2 - 3x + 2) = 0$
$x_1 = 0$ or $x^2 - 3x^2 + 2 = 0 \quad (a = 1, b = -3, \text{ and } c = 2)$

$S = \dfrac{-b}{a} \Rightarrow S = \dfrac{-(-3)}{1} \Rightarrow S = 3$

$P = \dfrac{c}{a} \Rightarrow P = \dfrac{2}{1} \Rightarrow P = 2$

Quick test: $+1 \quad +2$ $\quad x_1 = 1$ and $x_2 = 2$

$x_1 = 0, x_2 = 1$, and $x_3 = 2$
$x_1 + x_2 + x_3 = 0 + 1 + 2 = 3$

14. Given the equation $\sqrt{x + 10} = x + 4$, $x \in \mathbb{R}$. The sum of all possible solutions is:

a) -1
b) -2
c) -3
d) -4
e) None of the above.

Solution: a

$\sqrt{x + 10} = x + 4$
$(\sqrt{x + 10})^2 = (x + 4)^2$
$x + 10 = x^2 + 8x + 16$
$x^2 + 7x + 6 = 0 \quad (a = 1, b = 7, \text{ and } c = 6)$

$S = \dfrac{-b}{a} \Rightarrow S = \dfrac{-7}{1} \Rightarrow S = -7$

$P = \dfrac{c}{a} \Rightarrow P = \dfrac{6}{1} \Rightarrow P = 6$

Quick test: $-1 \quad -6$
$-2 \quad -3$

$x = -1$
check: $\sqrt{-1 + 10} = -1 + 4$
$3 = 3 \rightarrow$ True.
or $x = -6$
check: $\sqrt{-6 + 10} = -6 + 4$
$2 = -2 \rightarrow$ False.

The sum of all possible solutions is - 1.

15. Let x_1 and x_2 be the solutions to $x^2 - 2x - 1 = 0$. Consider the equation:

I. $x_1 + x_2 = -2$
II. $x_1 . x_2 = -1$
III. $x_1^2 + x_2^2 = 6$
IV. $x_1^3 + x_2^3 = 1$

Then:
a) Only II and III are correct.
b) Only I and III are correct.
c) Only II and III are correct.
d) Only II, III, and IV are correct.
e) None of the above.

Solution: a

$x^2 - 2x - 1 = 0 \quad (a = 1, b = -3, \text{ and } c = -10)$

$x_1 + x_2 = \dfrac{-b}{a} = \dfrac{-(-2)}{1} = 2$

$x_1 . x_2 = \dfrac{c}{a} = \dfrac{-1}{1} = -1$

$(x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$
$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2$
$x_1^2 + x_2^2 = 2^2 - 2(-1)$
$x_1^2 + x_2^2 = 6$
$\left(x_1 + x_2\right)^3 = x_1^3 + 3x_1^2 . x_2 + 3x_1 . x_2^2 + x_2^3$
$x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1^2 . x_2 - 3x_1 . x_2^2$
$x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2^2)$
$x_1^3 + x_2^3 = 2^3 - 3(-1)(2)$
$x_1^3 + x_2^3 = 14$

Thus, only II and III are correct.

16. Solve: $6x - 5 \geq 3x + 4$. The solution is:

a) $S = \{x \in \mathbb{R} \,/\, x \geq 3\}$
b) $S = \{x \in \mathbb{R} \,/\, x < 3\}$
c) $S = \{x \in \mathbb{R} \,/\, x \geq 4\}$
d) $S = \{x \in \mathbb{R} \,/\, x < 4\}$
e) None of the above.

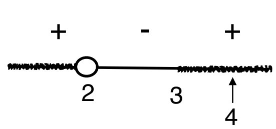
Solution: a

$6x - 5 \geq 3x + 4$
$6x - 3x \geq 4 + 5$
$3x \geq 9$
$x \geq 3$
Thus, $S = \{x \in \mathbb{R} \,/\, x \geq 3\}$.

17. Solve: $\dfrac{x-3}{x-2} > 0$. The solution is:

a) $S = \{x \in \mathbb{R} \,/\, x < -2 \text{ or } x > 3\}$
b) $S = \{x \in \mathbb{R} \,/\, x \leq -2 \text{ or } x \geq 3\}$
c) $S = \{x \in \mathbb{R} \,/\, 2 < x < 3\}$
d) $S = \{x \in \mathbb{R} \,/\, 2 \leq x \leq 3\}$
e) None of the above.

Solution: e



$\dfrac{x-3}{x-2} > 0$

$S = \{x \in \mathbb{R} \,/\, x < 2 \text{ or } x > 3\}$

Thus, none of the above.

18. Solve: $\dfrac{x^2 + 3x + 2}{x} > 0$. The solution is:

a) $S = \{x \in \mathbb{R} \,/\, x < -2 \text{ or } -1 < x < 0\}$
b) $S = \{x \in \mathbb{R} \,/\, -2 < x < -1 \text{ or } x > 0\}$
c) $S = \{x \in \mathbb{R} \,/\, x \leq -2 \text{ or } -1 \leq x < 0\}$
d) $S = \{x \in \mathbb{R} \,/\, -2 \leq x \leq -1 \text{ or } x > 0\}$
e) None of the above.

Solution: b

$\dfrac{x^2 + 3x + 2}{x} > 0$
$x^2 + 3x + 2 = 0 \quad (a = 1, b = 3, \text{ and } c = 2)$

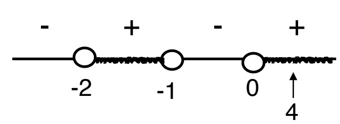
$S = \dfrac{-b}{a} \Rightarrow S = \dfrac{-(3)}{1} \Rightarrow S = -3$

$P = \dfrac{c}{a} \Rightarrow P = \dfrac{2}{1} \Rightarrow P = 2$

Quick test: $-1 \quad -2$

Then, $x^2 + 3x + 2 = (x+1)(x+2)$

Then, $\dfrac{(x+1)(x+2)}{x} > 0$



Thus, $S = \{x \in \mathbb{R} \,/\, -2 < x < -1 \text{ or } x > 0\}$.

19. Solve: $x^2 + x + 2 \leq 0$. The solution is:

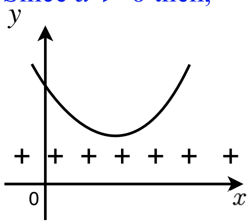
a) $S = \{x \in \mathbb{R} \,/\, x > 0\}$
b) $S = \{x \in \mathbb{R} \,/\, x < 0\}$
c) $S = \{x \in \mathbb{R} \,/\, x \leq 0\}$
d) $S = \mathbb{R}$
e) $S = \varnothing$ (There is no solution).

Solution: e

$x^2 + x + 2 \leq 0$
$x^2 + x + 2 = 0 \quad (a = 1, b = 1, \text{ and } c = 2)$
$\Delta = b^2 - 4ac$
$\Delta = (1)^2 - 4(1)(2)$
$\Delta = 1 - 8$

$\Delta = -7$

There is no solution.
Since $a > 0$ then,



Thus, $S = \varnothing$.

20. Solve: $\dfrac{x(x+2)}{(x+2)} < 0$. The solution is:

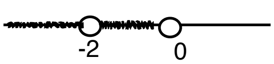
a) $S = \{x \in \mathbb{R} \,/\, x > 0\}$
b) $S = \{x \in \mathbb{R} \,/\, 0 < x < 2\}$
c) $S = \{x \in \mathbb{R} \,/\, x \leq 0 \text{ and } x \neq -2\}$
d) $S = \{x \in \mathbb{R} \,/\, x < -2 \text{ and } -2 < x < 0\}$
e) None of the above.

Solution: d

$\dfrac{x(x+2)}{(x+2)} < 0$

Existence Condition: $x + 2 \neq 0 \Rightarrow x \neq -2$

If $x \neq -2$, then $\dfrac{x(x+2)}{(x+2)} = x < 0$



Thus, $S = \{x \in \mathbb{R} \,/\, x < -2 \text{ and } -2 < x < 0\}$

PART 2: SOLUTIONS

Consulting

Name:______________________________ **Age:**_____ **Id:**________________ **Course:**__________

Multiple-Choice Answers

Questions	A	B	C	D	E
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					

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	Points	Max
Multiple Choice		100
Extra Points		25
Consulting		10
Age Points		25
Total Performance		160
Grade		A

Extra Questions

21. Solve: $x^4 + 9x^2 - 10 = 0\ ;\ x \in \mathbb{R}$.

Solution: $S = \{-1\ ,\ 1\}$

$y = x^2$

$y^2 + 9y - 10 = 0 \quad (a = 1, b = -15, \text{ and } c = -16)$

$$S = \frac{-b}{a} \Rightarrow S = \frac{-9}{1} \Rightarrow S = 9$$

$$P = \frac{c}{a} \Rightarrow P = \frac{-10}{1} \Rightarrow P = -10$$

Quick test:
$+1 \quad -10$
$+2 \quad -5$

$y = 1$ or $y = -10$

$x^2 = 1 \qquad x^2 = -10$

$x = \pm 1 \qquad \nexists\, x \in \mathbb{R}$

Thus, $S = \{-1\ ,\ 1\}$

22. Solve: $\frac{9}{x} + x \le -6\ ;\ x \in \mathbb{R}$.

Solution:

$$\frac{9}{x} + x + 6 \le 0$$

$$\frac{9 + x^2 + 6x}{x} \le 0$$

$$\frac{(x+3)^2}{x} \le 0$$

Since $(x+3)^2 \ge 0$. Then,

Case 1: $x + 3 = 0 \Rightarrow x = -3$ OK

Check $\frac{9}{-3} + (-3) \le -6 \Rightarrow -3 - 3 \le -6 \Rightarrow$
$-6 \le -6$

Case 2: $(x+3)^2 > 0$

$$\frac{(x+3)^2}{x} \le 0 \Rightarrow x < 0$$

Thus, $S = \{x \in \mathbb{R} \,/\, x < 0 \text{ or } x = -3\}$

23. Let x_1 and x_2 be the solution of the equation $x^2 - 2x - 6 = 0$.

Calculate $\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{x_1^2 + x_2^2}{x_1 x_2}$.

Solution: $\frac{x_1}{x_2} + \frac{x_2}{x_1} = -\frac{8}{3}$

$(x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$
$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2$

Then, $\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{(x_1 + x_2)^2 - 2x_1x_2}{x_1x_2}$

$x_1 + x_2 = \frac{-b}{a} \Rightarrow x_1 + x_2 = \frac{-(-2)}{1} = 2$

$x_1 \,.\, x_2 = \frac{c}{a} \Rightarrow x_1 \,.\, x_2 = \frac{-6}{1} = -6$

Thus, $\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{2^2 - 2(-6)}{(-6)}$

$\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{4 + 12}{-6}$

$\frac{x_1}{x_2} + \frac{x_2}{x_1} = -\frac{8}{3}$

24. Solve: $\frac{1-x}{2} = \frac{x-5}{3} + 1 \,;\, x \in \mathbb{R}$.

Solution: $S = \{\frac{7}{5}\}$

$\frac{1-x}{2} = \frac{x-5}{3} + 1$

Multiply all of the equation by 6.

$3(1-x) = 2(x-5) + 6$
$3 - 3x = 2x - 10 + 6$
$-3x - 2x = -4 - 3$
$-5x = -7$
$x = \frac{7}{5}$

25. Find the value of the integral below. If you're having trouble solving the integral, draw a circle as your answer to receive full credit.

$I = \int_0^1 x\,dx$

Solution: $\frac{1}{2}$

$I = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$

or